# Aperture of counter telescopes for parallel pairs of particles



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AUER, S., *et al.*, 1968, *Space Res.*, **8,** 606–16.
AUER, S., and SITTE, K., 1968, *Earth and Plan. Sci. Lett.*, **4,** 178–83.
FRIICHTENICHT, J. F., 1964, *Nucl. Instrum. Meth.*, **28,** 70–8.
HANSEN, D. O., 1968, *Appl. Phys. Lett.*, **13,** 89–91.
SHELTON, H., HENDRICKS, C. D., and WUERKER, R. F., 1960, *J. appl. Phys.*, **31,** 1243–6.

# Aperture of counter telescopes for parallel pairs of particles

**Abstract.** The difference between the aperture of a counter telescope for single particles and for pairs is explained. Expressions are given for both apertures and the significance of the difference for the interpretation of underground data is indicated.

In a study of groups of cosmic ray muons penetrating underground (Barton 1968) it was necessary to calculate the effective aperture of a telescope for parallel pairs of particles. The details of the calculation were omitted from that paper but, since the concept has given rise to some misunderstanding (Castagnoli *et al.* 1969, Bibilashvili private communication), it seems useful to explain it more clearly.

Following the notation of Stern (1960), the aperture of a counter telescope for single particles can be defined as $A_1(\rho)$, so that the counting rate $R_1$ for a particle intensity $I = I_0 \cos^{\rho}\theta$ is given by $R_1 = A_1(\rho) I_0$.

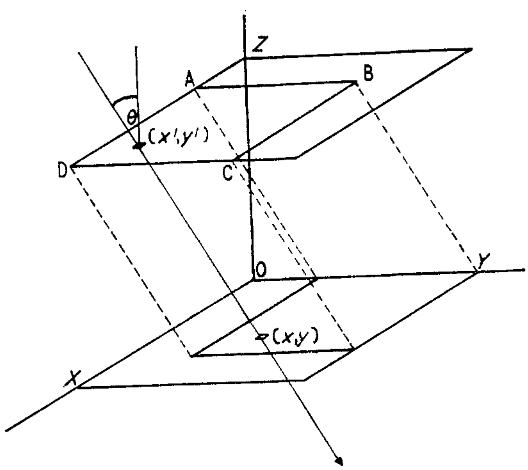


Figure 1. Telescope geometry.

If the telescope has two rectangular counters of dimensions $X$ and $Y$ at a separation $Z$ (figure 1), then, again following Stern,

$$A_1(\rho) = \frac{1}{Z^2}\int_0^X\int_0^Y\int_0^X\int_0^Y \cos^{\rho+4}\theta \, dx' \, dy' \, dx \, dy$$

where

$$\cos\theta = \frac{Z}{\{Z^2+(x-x')^2+(y-y')^2\}^{1/2}}.$$

Stern gives a mathematical transformation for evaluating this expression but, if $Z$ is not much smaller than $X$ or $Y$, it can also be computed by direct numerical integration.

Now consider the response to parallel pairs of particles. The pair counting rate $R_2$ is given by $R_2 = A_2(\rho)I_p$ provided the aperture and pair intensity are defined consistently. The definition of $I_p$ was given in the previous paper as the number of pairs per steradian crossing a horizontal area of one square metre in a vertical direction per unit time. Its dimensions are $m^{-4}\ sr^{-1}\ s^{-1}$. If the first particle traverses the counter as shown in figure 1, the second must fall within the area ABCD and the available area perpendicular to the direction of the pair is the area ABCD multiplied by $\cos\theta$. Hence the aperture for pairs is:

$$A_2(\rho) = \frac{1}{Z^2}\int_0^X\int_0^Y\int_0^X\int_0^Y \cos^{\rho+5}\theta(X-|x-x'|)(Y-|y-y'|) \, dx' \, dy' \, dx \, dy.$$

This formula should be correct provided the dimensions of the apparatus are small compared with the average separation between the particles; that is, the decoherence curve for the pairs is relatively flat.

As an example of the effect of this formula, the results of Chaudhuri and Sinha (1963) can be considered. The dimensions of their apparatus were $X = 0{\cdot}75$ m, $Y = 0{\cdot}30$ m, $Z = 0{\cdot}85$ m, and we assume that $\rho \simeq 2$ for single particles and $\rho \simeq 4$ for pairs at the relevant depth. The computed values are

$$A_1(2) = 0{\cdot}0502\ \mathrm{m^2\ sr}$$
$$A_2(4) = 0{\cdot}00522\ \mathrm{m^4\ sr}.$$

Their observed rate of $0{\cdot}11 \pm 0{\cdot}02$ per day therefore yields a vertical pair intensity of $21 \pm 4\ m^{-4}\ sr^{-1}\ d^{-1}$; Castagnoli *et al.* plot this point at $6 \pm 1{\cdot}2$, presumably through using a different expression for the aperture.

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Barton, J. C., 1968, *J. Phys.* A: *Gen. Phys.*, **1**, 43–54.
Castagnoli, C., Picchi, P., and Verri, G., 1969, *Nuovo Cim.*, **61B**, 290–301.
Chaudhuri, N., and Sinha, M. S., 1963, *Proc. Int. Conf. on Cosmic Rays, Jaipur*, 1963, (Bombay: Commercial Printing Press), pp. 106–10.
Stern, D., 1960, *Nuovo Cim. Suppl.*, **16**, 153–8.